B31.3 302.3.5(d)  
“When the computed stress range varies”  
– applying existing B31.3 rules in CAESAR II

...and a new piping code proposal:  
Allowable Stress for Wave Damage

B31.3 Paragraph 302.3.5(d)  
Allowable Displacement Stress Range $S_A$

When the computed stress range varies, whether from thermal expansion or other conditions, $S_A$ is defined as the greatest computed displacement stress range. The value of $S_A$ in such cases can be calculated by eq. (1d):

$$ N = N_0 + \sum_{i=1}^{\infty} \left( N_i / F_i \right) $$

where:
- $N_0$ = number of cycles of maximum computed displacement stress range, $S_0$
- $N_i$ = number of cycles associated with displacement stress range, $S_i$
- $F_i = S_i / S_0$
- $S_i$ = any computed displacement stress range smaller than $S_0$
Agenda

- Fatigue – Definitions and Use in B31
- Accumulated Damage & Miner’s Rule
- Equation (1d)
- Applying (1d)
- Using CAESAR II Fatigue Curve and Accumulated Damage to Satisfy (1d)
- A Worked Example
- A look at a Proposed Code addition providing High Cycle Fatigue Assessment of Piping Systems
Fatigue – a Definition*

- Fatigue is the progressive and localized structural damage that occurs when a material is subjected to cyclic loading. The nominal maximum stress values are less than the ultimate tensile stress limit, and may be below the yield stress limit of the material.

- Fatigue occurs when a material is subjected to repeated loading and unloading. If the loads are above a certain threshold, microscopic cracks will begin to form at the surface. Eventually a crack will reach a critical size, and the structure will suddenly fracture. The shape of the structure will significantly affect the fatigue life; square holes or sharp corners will lead to elevated local stresses where fatigue cracks can initiate. Round holes and smooth transitions or fillets are therefore important to increase the fatigue strength of the structure.

* from: http://en.wikipedia.org/wiki/Fatigue_(material)

Fatigue Assessment*

- Fatigue assessment [in the ABS Guide for the Fatigue Assessment of Offshore Structures 2004] relies on the characteristic S-N curve to define fatigue strength under constant amplitude stress and a linear damage accumulation rule (Palmgren-Miner) to define fatigue strength under variable amplitude stress.

Fatigue in B31

- In his 1947 paper*, A.R.C. Markl adopted the following general formula to reflect his fatigue test results:

\[
S_N = \frac{240,000}{\sqrt{f}}
\]

- Where \(S_N\) (in psi) is the endurance strength in terms of the number \(N\) of cycles of complete reversal producing failure.

- This is an endurance curve

* "Fatigue Tests of Welding Elbows and Comparable Double-Miter Bends" (Transactions of ASME Volume 69)

Fatigue in B31

- This S-N curve is expressed in the formula for \(S_A\), the allowable displacement stress range (B31.3 Eqn.(1a)):

\[
S_A = f \left(1.25S_e + 0.25S_h\right)
\]

Where, in Eqn.(1c):

\[
f = 6.0(N)^{-0.2}
\]

Old ASME II Part D S-N curve:

\[\text{Old ASME II Part D S-N curve:}\]

\[\text{f from B31.3:}\]
Fatigue in B31

B31.3 paragraph 302.3.5(d) states that the computed displacement stress range, $S_E$, shall not exceed the allowable displacement stress range, $S_A$; or:

$$S_E \leq 6.0(N)^{-0.2}(1.25S_c + 0.25S_a)$$

Compare with Markl:

$$S_E \leq S_N = 245,000(N)^{-0.2}$$

Or, compare a normalized $f$ with a normalized polished bar curve ($s$) from the current ASME II-D*:

* Here, normalized means the value equals 1.0 at 10,000 cycles
Fatigue stress is a random process. Stress ranges in the long-term process form a sequence of dependent random variables, $S_i; i = 1, N_T$. For purposes of fatigue analysis and design, it is assumed that $S_i$ are mutually independent. The set of $S_i$ can be decomposed and discretized into $J$ blocks of constant amplitude stress:

* from: COMMENTARY ON THE GUIDE FOR THE FATIGUE ASSESSMENT OF OFFSHORE STRUCTURES

<table>
<thead>
<tr>
<th>Stress Range $S_i$</th>
<th>Number of Cycles $n_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_1$</td>
<td>$n_1$</td>
</tr>
<tr>
<td>$S_2$</td>
<td>$n_2$</td>
</tr>
<tr>
<td>$S_3$</td>
<td>$n_3$</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>$S_{J-1}$</td>
<td>$n_{J-1}$</td>
</tr>
<tr>
<td>$S_J$</td>
<td>$n_J$</td>
</tr>
</tbody>
</table>
The Palmgren-Miner Rule defines Fatigue Damage*

- Applying the Palmgren-Miner linear cumulative damage hypothesis to the block loading of the preceding table, cumulative fatigue damage, $D$, is defined as:

$$D = \sum \frac{N_i}{N_i}$$

where $N_i$ is the number of cycles to failure at stress range $S_i$, as determined by the appropriate S-N curve.

- Failure is then said to occur if:

$$D > 1.0$$

* ibid

An example of accumulated damage

- For example:

<table>
<thead>
<tr>
<th>Given</th>
<th>Find</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_i$</td>
<td>$N_i$</td>
</tr>
<tr>
<td>A</td>
<td>$N_1$</td>
</tr>
<tr>
<td>C</td>
<td>$N_2$</td>
</tr>
</tbody>
</table>

- If $S_{ij}$ is stress range, use $S_{ij}/2$ as stress amplitude

$$D = \frac{N_1}{B} + \frac{N_2}{D}$$

Note: Old polished bar fatigue curve is used here only to demonstrate the process, do not use this curve in analysis.
Limitations Miner's Rule*

Though Miner's rule is a useful approximation in many circumstances, it has several major limitations:

- It fails to recognize the probabilistic nature of fatigue and there is no simple way to relate life predicted by the rule with the characteristics of a probability distribution.
- There is sometimes an effect in the order in which the reversals occur. In some circumstances, cycles of low stress followed by high stress cause more damage than would be predicted by the rule.

* from: http://en.wikipedia.org/wiki/Miner%27s_rule#Miner%27s_rule
B31.3 Paragraph 302.3.5(d)
Allowable Displacement Stress Range $SA$

When the computed stress range varies, whether from thermal expansion or other conditions, $S_a$ is defined as the greatest computed displacement stress range. The value of $N$ in each case can be calculated by Eq. (1d):

$$N = N_1 + \sum_{i=2}^n N_i$$ for $i = 1, 2, ..., n$

Where:
- $N_1 = \text{number of cycles at maximum computed displacement stress range, } S_a$
- $N_i = \text{number of cycles at displacement stress range, } S_i$
- $S_a = \text{any computed displacement stress range smaller than } S_a$

Deriving Equation (1d)

- Convert smaller stress ranges into equivalent cycles for the maximum stress range.
- Evaluate the largest calculated expansion stress range against an adjusted allowable limit

Terms:
- $S_E = \text{maximum stress range}$
- $S_i = \text{each smaller stress range}$
- $N_E = \text{cycles at } S_E$
- $N_i = \text{cycles at } S_i$
- $N_E \text{ allowed} = \text{cycles allowed at } S_E$
- $N_i \text{ allowed} = \text{cycles allowed at } S_i$
- $N_i \text{ equivalent} = \text{cycles at } S_E$
- $k = \text{Markl’s material constant}$
Deriving Equation (1d) 2/4

Markl says:
- \( S_{\text{allowed}} = kN^{-0.2} \)
  - or, solving for \( N \):
- \( N_{\text{allowed}} = \left(\frac{k}{S_i}\right)^5 \)

Terms:
- \( S_E = \text{maximum stress range} \)
- \( S_i = \text{each smaller stress range} \)
- \( N_E = \text{cycles at } S_E \)
- \( N_i = \text{cycles at } S_i \)
- \( N_{E \text{ allowed}} = \text{cycles allowed at } S_E \)
- \( N_{i \text{ allowed}} = \text{cycles allowed at } S_i \)
- \( N_{i \text{ equivalent}} = \text{cycles at } S_E \)
- \( k = \text{Markl’s material constant} \)

Deriving Equation (1d) 3/4

The ratio of actual cycles to allowed cycles could be used to determine the number of equivalent cycles for the maximum stress range

- \( \frac{N_i}{N_{i \text{ allowed}}} = \frac{N_{i \text{ equivalent}}}{N_{E \text{ allowed}}} \); or
- \( N_{i \text{ equivalent}} = N_i \cdot \frac{N_{E \text{ allowed}}}{N_{i \text{ allowed}}} \)
- \( N_{i \text{ equivalent}} = N_i \cdot \left(\frac{k}{S_E}\right)^5 \cdot \left(\frac{k}{S_i}\right)^5 = N_i \left(\frac{S_i}{S_E}\right)^5 \)

Terms:
- \( S_E = \text{maximum stress range} \)
- \( S_i = \text{each smaller stress range} \)
- \( N_E = \text{cycles at } S_E \)
- \( N_i = \text{cycles at } S_i \)
- \( N_{E \text{ allowed}} = \text{cycles allowed at } S_E \)
- \( N_{i \text{ allowed}} = \text{cycles allowed at } S_i \)
- \( N_{i \text{ equivalent}} = \text{cycles at } S_E \)
- \( k = \text{Markl’s material constant} \)
Deriving Equation (1d)

- with:
  \[ N_{i\text{equivalent}} = N_i \left( \frac{S_i}{S_E} \right)^5 \]

- letting:
  \[ r_i = \frac{S_i}{S_E} \]

- gives:
  \[ N = N_E + \sum (r_i^5 N_i) \]

When the computed stress range varies, whether from thermal expansion or other conditions, \( S_E \) is defined as the greatest computed displacement stress range. The value of \( N \) in such cases can be calculated by eq. (1d):

\[ N = N_E + \sum \left( c_i N_i \right) \text{ for } i = 1, 2, \ldots, n \]  

(1d)

where

- \( N_c \) = number of cycles of maximum computed displacement stress range, \( S_E \)
- \( N_i \) = number of cycles associated with displacement stress range, \( S_i \)
- \( r_i = \frac{S_i}{S_E} \)
- \( S_i \) = any computed displacement stress range smaller than \( S_E \)

B31.3 : COUNTING CYCLES
A Note on Counting Cycles – Rainflow Counting


The algorithm (Similar to ASME VIII-2 Annex 5-B)

1. Reduce the time history to a sequence of (tensile) peaks and (compressive) troughs.
2. Rotate this sheet clockwise 90° (earliest time to the top).
3. Each tensile peak is imagined as a source of water that “drips” down the pagoda.
4. Count the number of half-cycles by looking for terminations in the flow occurring when either:
   1. It reaches the end of the time history;
   2. It merges with a flow that started at an earlier tensile peak; or
   3. It flows opposite a tensile peak of greater magnitude …

5. Repeat step 5 for compressive troughs.
6. Assign a magnitude to each half-cycle equal to the stress difference between its start and termination.
7. Pair up half-cycles of identical magnitude (but opposite sense) to count the number of complete cycles. Typically, there are some residual half-cycles.

See also: TD/12
Counting Cycles – Example

- Given the stress history below, determine the total number of cycles for each stress range
- Note: Start = End = 0

Counting Cycles – Example

- Shift to start with largest stress
Counting Cycles – Example

- Starting with the maximum stress and always moving to the right, track the path to the lowest stress. Then, track the path back to the maximum.
- The path need not be contiguous.

Continue counting
Counting Cycles – Example

- Continue counting

Counting Cycles – Example

- Continue counting
Counting Cycles – Example

- Continue counting
Counting Cycles – Example

- Continue counting

© Intergraph 2015
Counting Cycles – Example

Continue counting

Counting Cycles – Summary

<table>
<thead>
<tr>
<th>Set</th>
<th>Count</th>
<th>Range</th>
<th>Max</th>
<th>Min</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>52</td>
<td>48</td>
<td>-4</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>46</td>
<td>44</td>
<td>-2</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>42</td>
<td>42</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>36</td>
<td>38</td>
<td>2</td>
</tr>
<tr>
<td>5</td>
<td>1</td>
<td>34</td>
<td>38</td>
<td>4</td>
</tr>
<tr>
<td>6</td>
<td>1</td>
<td>20</td>
<td>36</td>
<td>16</td>
</tr>
<tr>
<td>7</td>
<td>1</td>
<td>18</td>
<td>36</td>
<td>18</td>
</tr>
<tr>
<td>8</td>
<td>1</td>
<td>10</td>
<td>14</td>
<td>4</td>
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<td>9</td>
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<td>8</td>
<td>26</td>
<td>18</td>
</tr>
<tr>
<td>10</td>
<td>1</td>
<td>6</td>
<td>26</td>
<td>20</td>
</tr>
<tr>
<td>11</td>
<td>1</td>
<td>6</td>
<td>10</td>
<td>4</td>
</tr>
<tr>
<td>12</td>
<td>1</td>
<td>2</td>
<td>30</td>
<td>28</td>
</tr>
<tr>
<td>13</td>
<td>1</td>
<td>2</td>
<td>24</td>
<td>22</td>
</tr>
</tbody>
</table>
Another quick example

APPLYING (1d)
CAESAR II fatigue evaluation

- CAESAR II offers a more complete fatigue evaluation utilizing cumulative damage as calculated by the Miner’s Rule
- A fatigue curve must be provided to relate the stress (amplitude) to the allowed number of cycles, along with
- The expected number of cycles (rather than the “f” associated with that number of cycles)
- Where do we collect this S-N fatigue curve?

Using Equation (1d)

- This is difficult to apply!
  - The maximum computed displacement stress range, $S_e$, at any node can be set by any one of the several calculated stress ranges
  - The individual (lesser) stress ranges, $S_i$, vary as well

When the computed stress range varies, whether from thermal expansion or other conditions, $S_e$ is defined as the greatest computed displacement stress range. The value of $N$ as such could be calculated by eq. (1d):

$$N = N_1 + \sum_{i=1}^{n} N_i$$

where

- $N_1$ = number of cycles of maximum computed displacement stress range, $S_e$
- $N_i$ = number of cycles associated with displacement stress range, $S_i$
- $S_e$ = any computed displacement stress range smaller than $S_e$
Example: Pump manifold, analyze all hot and one spared pump.

This gives 4 operating states and 10 expansion ranges:

Note that CAESAR II now automatically creates ("recommends") all 10 ranges
**Different Maxima**

<table>
<thead>
<tr>
<th>Case</th>
<th>From</th>
<th>Code Stress</th>
<th>Allowable Stress</th>
<th>Design Code</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>50</td>
<td>11.13</td>
<td>304.70</td>
<td>150</td>
</tr>
<tr>
<td>2</td>
<td>50</td>
<td>11.13</td>
<td>304.70</td>
<td>150</td>
</tr>
<tr>
<td>3</td>
<td>50</td>
<td>11.13</td>
<td>304.70</td>
<td>150</td>
</tr>
<tr>
<td>4</td>
<td>50</td>
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<td>304.70</td>
<td>150</td>
</tr>
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<td>304.70</td>
<td>150</td>
</tr>
<tr>
<td>6</td>
<td>50</td>
<td>11.13</td>
<td>304.70</td>
<td>150</td>
</tr>
<tr>
<td>7</td>
<td>50</td>
<td>11.13</td>
<td>304.70</td>
<td>150</td>
</tr>
<tr>
<td>S2</td>
<td>50</td>
<td>11.13</td>
<td>304.70</td>
<td>150</td>
</tr>
</tbody>
</table>

**Several Ranges are Significant**

<table>
<thead>
<tr>
<th>Case</th>
<th>From</th>
<th>Code Stress</th>
<th>Allowable Stress</th>
<th>Design Code</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>50</td>
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<td>304.70</td>
<td>150</td>
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<td>150</td>
</tr>
<tr>
<td>3</td>
<td>50</td>
<td>11.13</td>
<td>304.70</td>
<td>150</td>
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<tr>
<td>4</td>
<td>50</td>
<td>11.13</td>
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<td>150</td>
</tr>
<tr>
<td>5</td>
<td>50</td>
<td>11.13</td>
<td>304.70</td>
<td>150</td>
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<tr>
<td>6</td>
<td>50</td>
<td>11.13</td>
<td>304.70</td>
<td>150</td>
</tr>
<tr>
<td>7</td>
<td>50</td>
<td>11.13</td>
<td>304.70</td>
<td>150</td>
</tr>
<tr>
<td>S2</td>
<td>50</td>
<td>11.13</td>
<td>304.70</td>
<td>150</td>
</tr>
</tbody>
</table>
Several Ranges are Significant

- Applying (1d) at Node 150:

<table>
<thead>
<tr>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>B</td>
<td>C</td>
<td>D</td>
<td>E</td>
<td>F</td>
<td>G</td>
</tr>
<tr>
<td>1</td>
<td>Load Case 13 sets the expansion stress range $S_E = 33.59$ MPa</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>Apply (1d): $N = N_E + \sum(r_i^2N_i)$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>$f$ changes from: $0.95$ (10,000 cycles) to: $0.84$ (19,081 cycles)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>Allowable stress drops by 12%</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>No other expansion stress ranges require evaluation for this node</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Checking Node 150

- Load Case 13 sets the expansion stress range $S_E = 33.59$ MPa
- Apply (1d): $N = N_E + \sum(r_i^2N_i)$
- $f$ changes from: $0.95$ (10,000 cycles) to: $0.84$ (19,081 cycles)
- Allowable stress drops by 12%
- No other expansion stress ranges require evaluation for this node
Fatigue in B31

- We will use B31.3 allowable displacement stress range equation (1a) as our fatigue curve in CAESAR II:

\[ S_a = f (1.25S_c + 0.25S_h) \]

- but

\[ f = 6.0(N)^{-0.2} \]

- so

\[ S_a = 6.0(N)^{-0.2}(1.25S_c + 0.25S_h) \]

- Equation (1b) is not as conservative but it includes the (perhaps varying) longitudinal stress due to sustained loads:

\[ S_a = 6.0(N)^{-0.2}[1.25(S_c+S_h) - S_L] \]
Fatigue Curve in CAESAR II

Excel Calculation

- Create a C3 fatigue curve to reflect Material
- Unit of psi (ksi)
- Sf = 20 ksi
- Sa = 20 ksi

\[
\begin{align*}
S_f & = 0.8 PAT-0.1 \\
S_a & = 1.2 \\
S_f & = \frac{S_a \cdot 0.25}{0.25} (N)
\end{align*}
\]

- N (x1000) f Sf (ksi)
- 0.01 1.20 36000
- 0.02 1.20 36000
- 0.01 1.02 30337
- 0.00 0.95 28258
- 0.00 0.98 26290
- 0.00 0.83 24305
- 0.00 0.69 20177
- 0.00 0.60 18999

CAESAR II Data File

Stored in the SYSTEM folder, CAESAR II will use these data to establish the fatigue curve in the analysis. Note the "Stress Multiplier" is set to 1.0 rather than the 0.5 found in other FAT files. We are indicating a range evaluation here, rather than the typical amplitude values in S-N curves.

Entering the Fatigue Curve (setting $S$)

1. In the Allowable Stress window, click on "Fatigue Curves" to open the dialog
2. "Read from file"
3. Select the fatigue file
Load Cases for Fatigue Evaluation (setting $N$)

- Set Stress Type to Fatigue
- Specify the Number of Cycles
- Open the Load Cycles column

Accumulated damage is calculated in the output processor

- Select all fatigue cases with the Cumulative Usage Report
- CAESAR II will calculate and sum all the selected $N_{demand}/N_{allowed}$ ratios
- OK, if the sum $D < 1$

$$D = \sum_{i=1}^{n} \frac{N_{i}}{N_{allowed}} < 1$$
A WORKED EXAMPLE
Comparing (1d) with CAESAR II fatigue evaluation

Worked Example

- Compare the cumulative damage approach (Markl fatigue curve) with the hand application of Equation (1d)
- CAESAR II model: SEVERAL STRAINS
  - A 3 meter cantilever of 4 inch STD A106B pipe
  - Anchored at one end (10)
  - Three imposed lateral displacements at the far end:
    - D1: 39mm, N: 14,500 cycles and N: 15,000 cycles
    - D2: 38mm, N: 14,500 cycles
    - D3: 36.5mm, N: 14,500 cycles
Calculate Stresses

What is the stress range (at node 10, the anchor) for each of the three imposed displacements:

<table>
<thead>
<tr>
<th>Displacement at 20 (mm)</th>
<th>Stress Range (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>D1</td>
<td>39.0</td>
</tr>
<tr>
<td>D2</td>
<td>38.0</td>
</tr>
<tr>
<td>D3</td>
<td>36.5</td>
</tr>
</tbody>
</table>

SE is the largest stress range. Here, $S_E = 150.73$ MPa (the first load set).

$N = N_E + \sum (r_i^5 N_i)$

$N = 14500 + 12732 + 10412 = 37644$

Calculate $N$ using (1d) (with 14,500 for each set)
Calculate $S_A$ (using 1a) and Evaluate

- $S_A = f(1.25S_c + 0.25S_h)$
- $f = 6.0(N)^{-0.2} = 6.0(37644)^{-0.2} = 0.73$
- $S_c = S_h = 137.892 \text{ MPa}$
- $S_A = 150.88 \text{ MPa}$

- $S_E = 150.73 \text{ MPa}$
- $S_E \leq S_A \checkmark$

Simple but Tedious

- No single expansion stress range will always produce the maximum stress range $S_E$
- Stress ratios will vary between load cases and vary from node to node
- An accounting headache!
The example fatigue curve reviewed earlier, MARKL AT 20KSI.FAT, matches the allowable stress range equation (1a)

The appropriate number of cycles was defined in the Load Case Editor. Note that the larger imposed displacement (D1) is entered twice, we will use the first entry, N=14500, now:

Select the proper set of loads to evaluate:
Using the CAESAR II Fatigue Curve & Accumulated Damage

View the results:

<table>
<thead>
<tr>
<th>Load Case</th>
<th>Cycles</th>
<th>From</th>
<th>Stress</th>
<th>Allowable</th>
<th>Usage</th>
<th>To</th>
<th>Stress</th>
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<th>Usage</th>
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<td>137.04</td>
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<td>0.98</td>
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<tr>
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<td>10</td>
<td>141.07</td>
<td>151.07</td>
<td>0.94</td>
<td>20</td>
<td>141.07</td>
<td>151.07</td>
<td>0.94</td>
</tr>
<tr>
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<td>481.48</td>
<td>0.95</td>
<td>20</td>
<td>451.48</td>
<td>481.48</td>
<td>0.95</td>
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</table>

Load Case Information | Results for Node 10 | Results for Node 20

Using the CAESAR II Fatigue Curve & Accumulated Damage

Node 10 details:

- Allowable Cycles comes from fatigue curve (given S, find N)
- Usage Ratio is (Cycles Required)/(Cycles Allowed)
- If the sum of ratios is < 1, fatigue is within limits
**Compare Results**

- The (1d) “hand” calculation resets the number of cycles used for the highest stress.
  - $S_E$ at Node 10 = max(150.73, 146.86, 141.07) = 150.73 MPa
  - $N_{equivalent} = 37644$, therefore $f = 0.729$
  - $S_E = 150.88$ MPa
  - $S_E < S_A$

- The CAESAR II Accumulated Damage report collects fatigue damage for each stress range.
  - $0.383 + 0.336 + 0.275 = 0.994 < 1$

**Reworked Example**

- Now, for the existing system and loads, adjust the number of cycles:

<table>
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<tr>
<th>Displacement at 20 (mm)</th>
<th>Stress Range (MPa)</th>
<th>$N_{previous}$</th>
<th>$N_{now}$</th>
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<td>150.73</td>
<td>14,500</td>
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<td>D2</td>
<td>38.0</td>
<td>146.86</td>
<td>14,500</td>
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<tr>
<td>D3</td>
<td>36.5</td>
<td>141.07</td>
<td>14,500</td>
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</table>
Recalculate N

- $S_E$ is the largest stress range. Here, $S_E = 150.73$ MPa (the first load set).

<table>
<thead>
<tr>
<th>i</th>
<th>Stress Range (MPa)</th>
<th>N</th>
<th>$r_i (=S_i/S_E)$</th>
<th>$r_i^5$</th>
<th>$r_i^5 N_i$</th>
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<tr>
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<td>150.73</td>
<td>15,000</td>
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<tr>
<td>1</td>
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<td>14,500</td>
<td>0.974</td>
<td>0.878</td>
<td>12,732</td>
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<td>2</td>
<td>141.07</td>
<td>14,500</td>
<td>0.936</td>
<td>0.718</td>
<td>10,412</td>
</tr>
</tbody>
</table>

- $N = N_E + \sum(r_i^5 N_i)$
- $N = 15000 + 12732 + 10412 = 38144$

Recalculate SA (1a) and Evaluate

- $S_A = f(1.25 S_c + 0.25 S_h)$
- $f = 6.0(N)^{-0.2} = 6.0(38144)^{-0.2} = 0.728$
- $S_c = S_h = 137.892$ MPa
- $S_A = 150.486$ MPa

- $S_E = 150.73$ MPa

- $S_E \leq S_A$ ✔
Using the CAESAR II Fatigue Curve & Accumulated Damage

Select the proper set of loads to evaluate:

From:

To:

Using the CAESAR II Fatigue Curve & Accumulated Damage

Node 10 details:
- With a higher cycle count, D1 Usage Ratio changes from 0.38 to 0.40
- Accumulated Damage now greater than 1.0
Conclusion

- "When the computed stress range varies" the CAESAR II fatigue evaluation (by accumulated damage) is equivalent to the application of B31.3 equation (1d).
- Accumulated damage is automatic in CAESAR II provided the proper fatigue curve is used and all expected cycle sets are counted.
- Accumulated damage evaluation in CAESAR II is simpler to apply than equation (1d).

Is All This Important?

- Remember, the cycle count is adjusted by the stress ratio to the 5th power:
  \[ N = N_x + \sum \left( \frac{\sigma_i}{K_F} \right)^5 N_i \]
- The multiplier drops rapidly with the ratio \( \frac{\sigma_i}{K_F} \):
  - ratio=0.8, increase \( N \) by 30\% \( N_i \)
  - ratio=0.6, use <10\% of \( N_i \)
Questions / Discussion?

A LOOK AT A PROPOSED CODE CHANGE
TO ACCOMMODATE WAVE LOADS

Appendix W
Source of this Material


- Related / companion documents
  - DNV-RP-C203 Fatigue Design of Offshore Steel Structures (with Commentary)

DEFINING ACCUMULATED FATIGUE DAMAGE
Remaining life for wave loads

- Consider the accumulated fatigue damage in groups:
  \[ D = \sum_{j=1}^{J} \frac{n_j}{N_{ij}} + \sum_{k=1}^{K} \frac{n_k}{N_{ik}} = d_j + d\_w \]

  where \( j \) represents the stress range-cycle pairs related to displacement loading and \( k \) represents stress range-cycle pairs related to wave loading.

- Calculate \( d_j \) as above to set remaining life (available damage) for wave loading:
  \[ d_j = \sum \frac{n_j}{N_{ij}} \]

- Since total damage must remain below 1.0, and with no fatigue design factor, the allowable fatigue damage for variable wave loading would then be:
  \[ d\_w = 1 - d_j \quad \text{(W-5)} \]

Wave loads are not discrete

- Fatigue damage due to wave loading is proportional to wave height (trough to peak). Wave height is random, not discrete; one would not count the number of cycles (\( N \)) for such random stress levels. Wave data often appears, instead, as a Probability Density Function (PDF).
Wave Terms

- From the Office of Naval Research

"Wavestats" by NOAA - NOAA UCAR COMET Program

Wave loads are random and continuous

- The fatigue curve will give the number of cycles to failure $N_i$ at stress level $s_i$ can be written as:
  $$N_{ci} = N(s_i)$$  (5.6)

- But now the number of cycles, $N_i$, grouped around stress level $s_i$, using the PDF, is based on the area under the PDF curve:
  $$N_i = N_c[f(s_i)\Delta s]$$  (5.7)

- Where: $N_{ci}$ is any reference life and $[f(s_i)\Delta s]$ is the fraction of the total number of cycles associated with $s_i$. 

Integrating…

- Substituting this $N_i$ into the summation above gives:

$$D_R = \sum \frac{N_i f_i(s_i) \Delta s}{N(s_i)}$$  \hspace{1cm} (5.8)$$

- $D_R$ is the total wave damage over a reference life $N_R$.

- The limit, as the group of stresses ($\Delta s$) around $s_i$ goes to zero:

$$D_R = N_A \int_0^{\infty} \frac{f(s)}{N(s)} \, ds$$  \hspace{1cm} (5.9)$$

Accumulated, random damage

- The number of cycles to failure $N(s)$ is set by the fatigue curve:

$$N(s) = A s^{-m}$$  \hspace{1cm} (5.10)$$

- Replacing $N(s)$ in (5.9) above, we now have the accumulated damage over a life $N_A$ as:

$$D_R = \frac{N_A}{A} \int_0^{\infty} s^{-m} f(s) \, ds$$  \hspace{1cm} (5.11)$$

- Again, $D_R$ is the (reference) damage associated with $N_A$ (reference) cycle life.
How can this random distribution of stress levels be quantified?

Assume that the stress level produced by wave load is directly proportional to wave height. Historic wave data for certain bodies of water (e.g., North Sea and Gulf of Mexico) show a Weibull distribution of the number of waves at a certain height.
Two-parameter Weibull Distribution

- Let $S$ be a random variable denoting a single stress range associated with wave height in a long-term wave history.
- Assume that $S$ has a two-parameter Weibull distribution. The probability that the random variable stress range, $S$, is less than or equal to a certain stress level $s$ is:

$$F_S(s) = P(S \leq s) = 1 - e^{-\left(\frac{s}{q}\right)^h} \quad (5.1)$$

- This is a cumulative distribution function
- $h$ and $q$ are the Weibull shape and Weibull scale parameters, respectively.

Weibull terms - Shape

- Here are some examples of the Weibull distribution:

```
f(occurrences, h, q, l)
f(t, 0.5, 100, 0)
f(t, 1, 100, 0)
f(t, 3, 100, 0)
```

- varying the Weibull parameters:
  - $h$: shape parameter
  - $q$: scale parameter
  - $l$: location parameter
Weibull terms - Scale

Here are some examples of the Weibull distribution:

\[ f(t) = \frac{h}{\lambda} \left(\frac{t}{\lambda}\right)^{h-1} e^{-\left(\frac{t}{\lambda}\right)^h} \quad \text{for} \quad t \geq 0, h > 0, \lambda > 0 \]

\[ f(t, 1, 10, 0) \]
\[ f(t, 1, 50, 0) \]
\[ f(t, 1, 100, 0) \]

Appendix W offers a default shape parameter

Here is the Weibull probability distribution where \( h = 1.0 \) (the value mentioned in Appendix W to represent the shape distribution for a typical sea state):

\[ f(\text{wave_height}, h, \frac{q_{\text{max}}}{\lambda}) \]

This plot shows that there are many, many more occurrences of low stress ranges (small waves) than there are high stress ranges (big waves).

In Appendix W, stress range is assumed directly proportional to wave height but keep in mind that \( h \) indicates Weibull shape parameter and not wave height.
Wave damage

Using the cumulative distribution function $F_s(s)$, the number of cycles at stress level $s$ is:

$$f_s(s) = \frac{ds}{dt}$$  \hspace{1cm} \text{(para. 5.3)}

With $F_s(s) = 1 - e^{-\left(\frac{h}{q}\right)^n}$:

$$f_s(s) = \left(\frac{\bar{s}}{q}\right)^n (\bar{s})^{n-1} e^{-\left(\frac{h}{q}\right)^n}$$  \hspace{1cm} \text{(5.17)}

Integrating (5.11), the damage at design life (replacing reference life $N_R$ with design life $N_d$) is:

$$D = \frac{N_d}{A} q^m \Gamma\left(\frac{m}{n} + 1\right)$$  \hspace{1cm} \text{(5.19)}

with the gamma function $\Gamma(.)$ defined as:

$$\Gamma(x) = \int_0^\infty t^{x-1} e^{-t} dt$$  \hspace{1cm} \text{(5.3)}
Evaluating the Gamma Function

\[ D = \frac{N_d}{A} q^m \Gamma \left( \frac{m}{h} + 1 \right) \]

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<th>1/hm/h</th>
<th>F1/hm/h</th>
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Setting the Weibull Scale parameter

- But the total damage calculation also requires the Weibull scale parameter \( q \):

\[ D = \frac{N_d}{A} q^m \Gamma \left( \frac{m}{h} + 1 \right) \quad (5.19) \]

- This parameter also appears in the cumulative distribution function:

\[ F_q(s) = P(S \leq s) = 1 - e^{-\left(\frac{s}{h}\right)^q} \quad (5.1) \]

  - This is the probability that a single stress level \( S \) is equal to or below a stress level \( s \).

- This function could be rewritten to determine the probability that a stress level \( S \) is above some value \( s \), as in:

\[ F_q(s) = P(S > s) = e^{-\left(\frac{s}{h}\right)^q} \]

© Intergraph 2015
Illustrating this probability

Probability Density Function of $s$

- Probability of stress $S \leq S_R$:
  $$P(S \leq S_R) = 1 - e^{-\left(\frac{S_R}{\theta}\right)^k}$$

- Probability of stress $S > S_R$:
  $$P(S > S_R) = e^{-\left(\frac{S_R}{\theta}\right)^k}$$

Setting the Weibull Scale parameter

- The probability of a stress level $S$ exceeding a reference stress level, $S_R$, is:
  $$P(S > S_R) = e^{-\left(\frac{S_R}{\theta}\right)^k}$$

- The “100 year storm” can be used to set this probability where the reference stress level is based on the “100 year storm” wave height.

- By definition, this wave height would be reached once in 100 years, or, in $N_w$ cycles.

- So the reference stress level – the stress associated with the 100 year storm height – will occur once every $N_w$ cycles:
  $$P(S > S_R) = e^{-\left(\frac{S_R}{\theta}\right)^k} = \frac{1}{N_w}$$

- Solving for the Weibull scale parameter:
  $$q = \frac{s_R}{\left(\ln(N_w)\right)^{1/k}}$$

- This $q$ is independent of the length of time (or cycles, $N_w$) considered.
Use ASME VIII-2 S-N welded fatigue data

- ASME Section VIII Division 2 Annex 3-F, paragraph 3-F.2 provides the number of allowed cycles for welded joints in equation (3-F.4).
- But equation (3-F.4) references an equivalent structural stress range rather than the B31.3 stress range defined in paragraph 319.
- Equation (W-1) includes additional adjustments provided in paragraph 5.5.5 of VIII-2 to produce the allowed number of cycles for a welded joint using the B31.3 expansion stress range formula:

$$N_{11} = \frac{f_a}{f_k} \left( \frac{C (f_{S-N} f_{N})}{f_k} \right)^{n_k} \tag{W-1}$$

3-F.2.2

The number of allowable design cycles for the welded joint fatigue curve shall be computed as follows. 
(a) The design number of allowable design cycles, N, can be computed from Equation (3-F.4) based on the equivalent structural stress range parameter, $\Delta S_{eq}$, determined in accordance with paragraph 5.5.5 of this Division. The constants C and k for use in Equation (3-F.4) are provided in Table 3-F.10. The lower 99% Prediction Interval $(~3\sigma)$ shall be used for design unless otherwise agreed to by the Owner-User and the Manufacturer.

$$N = \frac{1}{f_k} \left( \frac{f_{S-N} f_{N}}{\Delta S_{eq}} \right)^{\frac{1}{k}} \tag{3-F.4}$$
Reformulation

\[ N_{el} = \frac{f_d}{f_x} \left( \frac{C_f E f_{eq}}{S_{el e_x}} \right)^m \]  \hspace{1cm} (W-1)

- Re-writing (W-1) in the form \( N = aS^{-m} \), you can show:

\[ a = \frac{f_d}{f_x} \left( \frac{C_f E f_{eq}}{S_{el e_x}} \right)^m \]  \hspace{1cm} (W-9)

- So:

\[ N_{el} = a \cdot S_{el}^{-m} \]

Total wave damage

- Total wave damage is:

\[ D = \frac{N_d a^{-m}}{\pi} \Gamma \left[ \frac{m}{\pi} + 1 \right] \]

- Evaluating \( q \) with \( S_R = S_d \) (the B31.3 expansion stress range associated with the 100 year storm wave height), and \( N_d \) as the design number of cycles, the accumulated wave fatigue damage, \( D \), is:

\[ D = \frac{N_d a}{\pi} \left( \frac{S_{el}}{\sigma R / \xi} \right)^m \Gamma \left[ \frac{m}{\pi} + 1 \right] \]
Setting the allowed stress for the maximum wave damage

- This fatigue damage due to wave loads $d_w$ plus the fatigue damage from other sources $d_t$ cannot exceed 1.0. Therefore, $d_w$ is remaining life after other, non-wave displacement cycles ($d_t$)

$$D = d_w = 1 - d_t$$

- Solve for $S_w$ and set that as your “allowable maximum probable stress range during $N_d$ wave cycles”. $N_d$ is the number of design life cycles for the system (e.g., cycles over a 20 year life).

$$S_{aw} = \left( \frac{d_{aw}}{N_d} \right)^{\frac{1}{n}} \cdot \frac{\ln(N_d)}{\Gamma\left(\frac{n}{n-1}\right)}$$

(W-8)

- where:

$$a = \frac{f_y}{f_x} \left( \frac{CF_s B K}{E} \right)^m$$

(W-9)

All terms are defined

- So, as long as the stress range associated with the maximum expected wave height (e.g., the 100 year storm height) is below $S_{aw}$, fatigue failure is not predicted.

$$S_{aw} = \frac{1}{c_{ME}} \left( \frac{d_{aw}}{N_d} \right)^{\frac{1}{n}} \cdot \frac{\ln(N_d)}{\Gamma\left(\frac{n}{n-1}\right)}$$

(W-8)

- where:

$$a = \frac{f_y}{f_x} \left( \frac{CF_s B K}{E} \right)^m$$

(W-9)

$$d_w = 1 - d_t$$

(W-5)

- In this manner, the stress range need only be calculated for the 100 year storm wave height and the accumulate wave damage will be estimated using the Weibull stress range distribution.
Example

- Given
  - Units: Metric
  - Material: Ferritic Steel
  - T-bar: 9.525 mm
  - Stress range is less than yield
  - Pipe will be in seawater and will have no cathodic protection
Example

\[ a = \frac{f_I}{f_E} \left( \frac{C_F f_M k f_1}{T_E^k} \right)^m \]  

(W-9)

- Fatigue Improvement Factor (ASME VIII-2) \( f_I = 1.0 \)
- Environmental Correction Factor (Table W302.2) \( f_E = 3.0 \) (seawater with free corrosion)
- Welded Joint Fatigue Curve Coefficient (Table W302.1a) \( C_F = 14137 \)
- Fatigue Factor for stress ratio \( f_{M,k} = 1.0 \)
- Temperature correction factor \( f_t = 1.0 \)
- Effective component thickness (text in W302.1) \( T_E = 16 \)
- Welded Joint Fatigue Curve Exponent (Table W302.1a) \( m = 3.13 \)
- Fatigue strength thickness exponent (Table W302.1a) \( k = 0.222 \)

\[ a = \frac{1}{3} \left( \frac{14137 \cdot 1.1}{16^{0.222}} \right)^{3.13} \]

\[ a = 0.475 \times 10^{12} \]

Example

\[ d_w = 1 - d_t \]

- For this example, let the fatigue damage due to thermal stress with constant amplitude \( d_t = 0.80 \)

\[ d_w = 1 - 0.60 \]

\[ d_w = 0.40 \]
Example

- **Design Storm Wave height associated cycles** ($N_w$)
  
  \[ N_w = 3.156 \cdot 10^7 \cdot V_o \cdot L_w \]  
  \( \text{(W-6)} \)

  - Average zero-crossing frequency in Hertz (typical, see W302.2.1) $V_o=0.159$ (period of about 6 sec)
  - Design Storm Period of Occurrence in years, $L_w=100$

  \[ N_w = 3.156 \cdot 10^7 \cdot 0.159 \cdot 100 \]
  \[ N_w = 501.18 \cdot 10^6 \]

- **Design number of pipe stress cycles** ($N_d$)
  
  \[ N_d = 3.156 \cdot 10^7 \cdot V_o \cdot L_d \]  
  \( \text{(W-7)} \)

  - Average zero-crossing frequency in Hertz, $V_o=0.159$
  - Piping Cyclic Design Life in years, $L_d=20$

  \[ N_d = 3.156 \cdot 10^7 \cdot 0.159 \cdot 20 \]
  \[ N_d = 100.04 \cdot 10^6 \]

- **Example**

  \[ S_{aw} = \left( \frac{d_{aw}}{N_d} \right)^{\frac{1}{m}} \cdot \frac{\ln(N_w)}{\Gamma \left( \frac{m}{m} + 1 \right)} \]  
  \( \text{(W-8)} \)

  - Allowable Fatigue damage for variable Wave Loadings (above) $d_{aw}=0.40$
  - Adjusted S-N constant (above) $a=0.475 \cdot 10^{12}$
  - Design number of pipe stress cycles (above) $N_d=100.04 \cdot 10^6$
  - Welded Joint Fatigue Curve Exponent (Table W302.1a) $m=3.13$
  - Design Storm Wave height associated cycles (above) $N_w=501.18 \cdot 10^6$
  - Weibull stress range shape distribution parameter (typical, see W302.2.1) $h=1.0$
  - Gamma Function evaluation (Table W301 where $\left( \frac{m}{h} + 1 \right)$=4.14) $\Gamma(4.14)=7.17$

  - $\frac{m}{h} + 1 = \frac{3.13}{1} + 1 = 4.14$

  \[ S_{aw} = \left( \frac{0.40 - 0.475 \cdot 10^{12}}{100.04 \cdot 10^6} \right)^{\frac{1}{3.13}} \cdot \frac{\ln(501.18 \cdot 10^6)}{\Gamma(4.14)} \]  
  \[ = 119 \text{ MPa} \]
Example

- The computed maximum stress range due to wave motion – $S_{EW}$ – shall remain below the allowable maximum probable stress range – $S_{aw}$ – through the expected life of the system.

- Here:
  - $S_{EW}$ is calculated in accordance with B31.3 paragraph 319 for the maximum wave height
  - $S_{aw}$ is 119 MPa

- In this example, the calculated B31.3 expansion stress range caused by maximum probable wave height (trough to peak), $S_{EW}$, shall not exceed $S_{aw}$ (119 MPa).

- This $S_{aw}$ changes from node to node in the piping system

Other notes of interest in Appendix W

- Applies where the total number of significant cycles exceed 100,000.
- A significant cycle is a stress range that exceeds 20.7 MPa
- Appendix W does not address pressure cycling.
- Integral construction is recommended, fabricated components are not recommended
- An optional (bi-linear) fatigue curve is available for cycle counts above 10 million
- The design Sea State (setting the wave height, wave period and probability density) shall be specified by the owner
- This proposed appendix also has additional requirements regarding fluid service, materials, fabrication, examination and testing
B31.3 302.3.5(D) “WHEN THE COMPUTED STRESS RANGE VARIES” – APPLYING EXISTING B31.3 RULES IN CAESAR II

Questions / Comments?

Thank you